Problem 7

Use the preliminary test to decide whether the following series are divergent or require further testing. *Careful:* Do *not* say that a series is convergent; the preliminary test cannot decide this.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n^3+1}}$$

Solution

Take the limit of the summand as $n \to \infty$.

$$\begin{aligned}
\lim_{n\to\infty} \frac{(-1)^n n}{\sqrt{n^3+1}} &= \left[\lim_{n\to\infty} (-1)^n\right] \left(\lim_{n\to\infty} \frac{n}{\sqrt{n^3+1}}\right) \\
&= \left[\lim_{n\to\infty} (-1)^n\right] \left[\lim_{n\to\infty} \frac{n}{\sqrt{n^3\left(1+\frac{1}{n^3}\right)}}\right] \\
&= \left[\lim_{n\to\infty} (-1)^n\right] \left(\lim_{n\to\infty} \frac{n}{n^{3/2}\sqrt{1+\frac{1}{n^3}}}\right) \\
&= \left[\lim_{n\to\infty} (-1)^n\right] \left(\lim_{n\to\infty} \frac{1}{n^{1/2}\sqrt{1+\frac{1}{n^3}}}\right) \\
&= \left[\lim_{n\to\infty} (-1)^n\right] \left(\lim_{n\to\infty} \frac{1}{n^{1/2}}\right) \left(\lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^3}}}\right) \\
&= \left[\lim_{n\to\infty} (-1)^n\right] (0)(1) \\
&= 0
\end{aligned}$$

Since it's zero, no conclusion can be drawn. Further testing is needed.